FAU ERLANGEN-NÜRNBERG

Mathematics for Economists
Exercise Sheet 1 – Oct 9, 2022

Dr. Ricardo Correa da Silva

1.) Given $f(t) = 2t$, $g(t) = t - 1$ and $h(t) = t^2$. Form the compositions

a) $f(g(t))$	*b)* $f(h(t))$	*c)* $g(h(t))$
d) $g(f(t))$	*e)* $h(g(t))$	*f)* $h(f(t))$
g) $f(f(t))$	*h)* $g(g(t))$	*i)* $h(h(t))$
j) $f(g(h(t)))$	*k)* $g(f(h(t)))$	*l)* $h(g(f(t)))$

2.) Determine for the following functions $f : \mathbb{R}^2 \to \mathbb{R}$ the maximal domain $D_f \subseteq \mathbb{R}^2$, the range $W_f \subseteq \mathbb{R}$ and the zeros:

a) $f(x, y) = \sin(xy)$, *b)* $f(x, y) = \cos(xy)$,
c) $f(x, y) = \tan(xy)$, *d)* $f(x, y) = \cot(xy)$.

3.) (Inverse functions).

a) Examine the invertibility of the following functions and specify the image. If possible, determine the inverse function.

$$\begin{aligned} f_1 &: \mathbb{R} \to \mathbb{R}, & x &\mapsto 3x + 29, \\ f_2 &: [0, \infty) \to \mathbb{R}, & x &\mapsto 3x + 29, \\ g_1 &: \mathbb{Z} \to \mathbb{Z}, & x &\mapsto x^2, \\ g_2 &: \mathbb{Z} \to \mathbb{N}_0, & x &\mapsto x^2. \end{aligned}$$

The image of a function $f : X \to Y$ is the set of all values of Y which are taken by $f(X)$.

b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{1}{2}x^2 - x$ and $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = -2x + 3$. Determine the functions $h_1 = g \circ f$ and $h_2 = f \circ g$.

4.) (Continuity). Check whether the following functions are continuous or not.

1. $f_1 : \mathbb{R} \to \mathbb{R}$ with $f_1(x) = x^5 + 2x^3 - x^2 - 2$,
2. $f_2 : (\frac{1}{2}, \infty) \to \mathbb{R}$ with $f_2(x) = \begin{cases} 5 + \tan(\pi x) & x \in (\frac{1}{2}, 1), \\ x^2 + 2x + 2 & x \in [1, 3), \\ \frac{17}{x} & x \in [3, \infty). \end{cases}$

5.) (Matrices: addition and multiplication).

1. Let the following matrices be given:

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}.$$

   Check for every combination of two matrices whether the sum or product is defined and compute them, if possible. All possible combinations are $A+A$, $A+B$, $A+C$, $B+A$, $B+B$, $B+C$, $C+A$, $C+B$, $C+C$, AA, AB, AC, BA, BB, BC, CA, CB, CC.

2. Let $n \in \mathbb{N}$ and $A, B \in \mathbb{R}^{n\times n}$. Does the following equation always holds?

$$(A+B)^2 = A^2 + 2AB + B^2.$$

   $\big($For matrices we also write $A^2 = AA$, $A^3 = AAA$, $\cdots$, $A^n = \underbrace{A\cdots A}_{n-\text{times}}\big)$.

(Question of the week). Answer spontaneously before you check your answer! Which of the following terms is smaller than the other one:

$$1000^{1000} \text{ oder } 1001^{999}?$$